Zolotarev Branch-Guide Couplers

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***Abstract*—A new theory for the design of branch-guide couplers, which enables the designer to optimize the internal impedance level, is presented. This overcomes previous difficulties associated with design of branch-guide couplers for tight-coupling values. Results are demonstrated for couplings as tight as 1.5 dB and give exact agreement between computer predictions and measured results. The theory was used to design a complex matched power divider employing many branch-guide couplers of 14 different coupling values and gave practical results in close agreement with theory.**

Introduction

The PRECISE theory of branch-guide couplers given in [1] has been used as a basis for the accurate design of practical couplers [2]. The coupling of a branch-guide coupler may be predicted in practice within an error that is probably less than that caused by tolerance variations. Since the branch-guide coupler is also a very compact device, this makes its use very attractive for many applications where nonstandard coupling values, waveguide sizes, and frequency bands are specified. The coupler may be designed using a computer and manufactured from the dimensions so obtained with complete confidence that it will work without further empirical adjustments. The only exception that should be made to this statement is "providing that the main line impedances are not significantly different from unity, and the branch-guide impedances do not result in slots that are much wider than about half the main waveguide width."

However, examination of the tables published in [1] shows that for tight-coupling values, the normalized main line impedances become quite large, e.g., approximately 1.65 in the central region of the coupler for 3-dB coupling. Additionally, the central branch or branches have normalized impedances of approximately unity. This implies that 3-dB waveguide couplers could not be successfully designed directly from the values tabulated in [1]. The widths of the central branches would be so wide that they would have a tendency to overlap, or at best to leave only a thin metallic wall between branches. The situation becomes impossible if a coupling tighter than 3 dB is required, e.g., 0 dB, as in [3]. Here it is necessary to cascade two or three couplers, each individually having looser coupling than the overall coupling, e.g., two 8.34-dB couplers to obtain 3-dB coupling, or three 6-dB couplers to obtain 0-dB coupling.

The disadvantage of this technique of cascading couplers is that it gives a highly redundant nonoptimum design. Take for example the design of a 3-dB coupler for a bandwidth $\lambda_{g0}/\lambda_{g2}=1.3$. Here the design band extends from guide wavelengths λ_{g1} through λ_{g2}, with λ_{g0} being the midband guide wavelength given by

$$\lambda_{g0} = \frac{2\lambda_{g1}\lambda_{g2}}{\lambda_{g1}+\lambda_{g2}}. \tag{1}$$

The tables in [1] show that in order to obtain a directivity of 40 dB, then $n=6$ branches are required, but the central normalized main line impedance is then approximately 2. To overcome this, it is necessary to cascade two 8.34-dB couplers, but each will still need to have 6 branches since a 5-branch 8.34 coupler has a directivity of only 36 dB. Hence, the overall coupler will have 11 branches (assuming that the two adjacent branches of each 8.34-dB coupler are combined to form one branch). In addition to the length disadvantage, the first branch guide has a normalized impedance of 0.04, which gives a very narrow coupling slot in the smaller waveguide sizes (e.g., 0.016 in for WR90, 0.9×0.4-in internal dimensions). Note that these difficulties do not exist in the case of TEM proximity couplers, which have perfect directivity in theory.

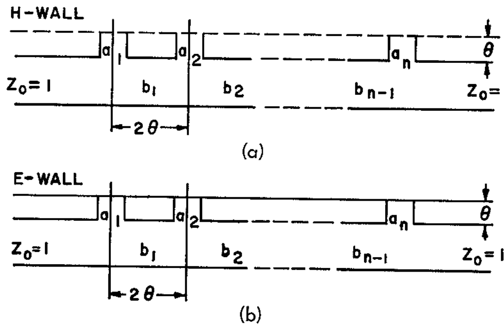

Fig. 1. (a) Even-mode two-port network. (b) Odd-mode two-port network.

One way to overcome this problem of designing waveguide branch-guide couplers having tight-coupling values might be to revert to older approximate design techniques, as summarized in [4]. However, the object of this paper is to present a new exact method that overcomes all of the difficulties outlined above. It enables the designer to vary the internal impedance levels and to compromise between these and the directivity and VSWR.

Discussion of Previous Theory

The even- and odd-mode prototype two-port networks of the branch-guide coupler are shown in Fig. 1. Each consists of a cascade of shunt or series stubs of electrical length θ spaced by main lines of twice this length. The theory presented in [1] is based on the generalized rational Chebyshev function

$$Q_{n-1}\left(\frac{x}{x_c}\right) = \frac{P_{n-1}\left(\frac{x}{x_c}\right)}{\sqrt{1-x^2}} \tag{2}$$

where

$$P_{n-1}\left(\frac{x}{x_c}\right) = \frac{1}{2}(1+\sqrt{1-x_c^2})T_{n-1}\left(\frac{x}{x_c}\right) - \frac{1}{2}(1-\sqrt{1-x_c^2})T_{n-3}\left(\frac{x}{x_c}\right) \tag{3}$$

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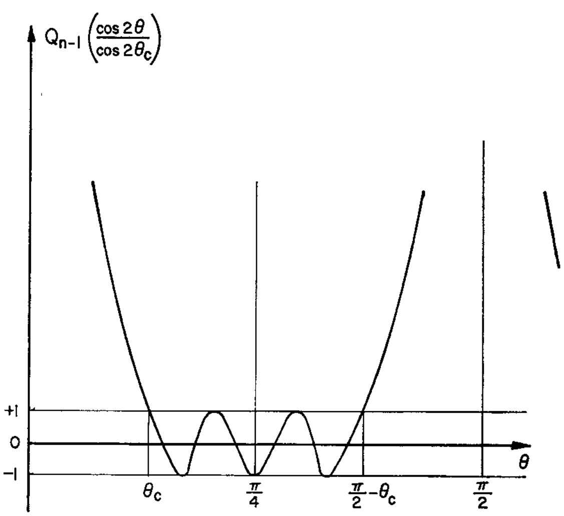

Fig. 2. Chebyshev function for case $n=7$.

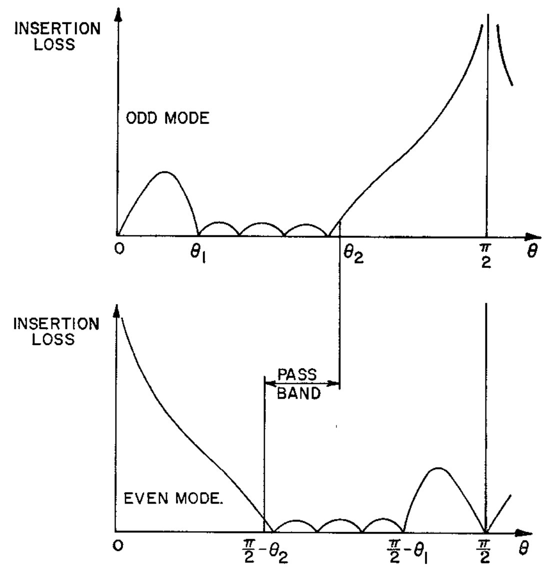

Fig. 3. Characteristics of odd- and even-mode branch-guide coupler networks designed using Zolotarev functions.

and T_n denotes the Chebyshev polynominal of the first kind of degree n. Substituting

$$x = \cos 2\theta \qquad x_c = \cos 2\theta_c \tag{4}$$

then Q_{n-1} is an equiripple function between θ_c and $(\pi/2)-\theta_c$ on the θ axis, as shown in Fig. 2. There are single-ordered poles at integral multiples of $\pi/2$.

The even- and odd-mode functions Γ_e/T_e and Γ_o/T_o (Γ denoting the reflection coefficient and T the transmission coefficient) are obtained by multiplying function (2) by $2jK \sin^2\theta$ and $2jK \cos^2\theta$, respectively. Here, to comply with the theory given in [1], the even-mode network is defined temporarily for the shunt stub case, which is matched at dc rather than the dual series stub case appropriate to waveguide branch-guide couplers. The above modifications to function (2) result in [1, eq. (25), (29)]. For example, multiplication by $2jK \sin^2\theta$ forces a zero at $\theta=0$, as required by the shunt stub even-mode network, and gives a function Γ_e/T_e having the correct generic form to represent an n-stub, $n-1$ main line network. Of course the Γ/T functions are no longer equiripple in the band $\theta_c<\theta<(\pi/2)-\theta_c$, but the sum and difference of Γ_e and Γ_o, representing the input reflection coefficient and directivity, respectively, are approximately equiripple.

The reason for choosing the function (2) as a starting point was to maximize the number of zeros in the design band. It also ensures that the zeros of both Γ_e and Γ_o coincide exactly.

New Theory

An alternative approach described in [5] is to make both Γ_e/T_e and Γ_o/T_o equiripple in the design band. If this is done then the zeros of Γ_e and Γ_o do not exactly coincide, and the performance of the coupler is slightly less optimum, as discussed in [5]. The zeros of Γ_e and Γ_o are not coincident because in general the zeros are not distributed with exact arithmetic symmetry about the midband $\theta=\pi/4$. However, small deviations in the zeros of Γ_e and Γ_o cause only slight loss in performance, as shown in [5] for the case of a three-branch coupler. This is true also for couplers of higher order, as demonstrated later in this paper.

The new and more flexible design method is in fact to make the Γ/T functions individually equiripple over the design band. A disadvantage of this procedure when [1] and [5] were written was that the explicit formula for the insertion loss function was not known. This has now been remedied by the appearance of the generalized rational Zolotarev functions described in [6]. The application to the synthesis of equiripple functions appropriate to a cascade of commensurate stubs spaced by double-length unit elements is given in greater detail in [7]. The insertion loss function for the network matched at dc (open-circuited shunt stubs or short-circuited series stubs) is given in [7, fig. 4 and eq. (4)], and will not be repeated here. The insertion loss function for the short-circuited series stub case is shown by the odd-mode response in Fig. 3. Now the even-mode response is complementary to the odd-mode response, i.e., it is obtained by the transformation

$$\theta \rightarrow \frac{\pi}{2} - \theta. \tag{5}$$

It is obvious that optimum performance will be obtained when the passbands of the odd- and even-mode networks coincide; that is, when $\theta_1+\theta_2=\pi/2$. We see, however, that in the general case the passband for the coupler extends from $(\pi/2)-\theta_2$ to θ_2, and θ_1 represents a free parameter that may be varied in order to change the impedance level within the coupler. We would expect the overall coupler performance to deteriorate if θ_1 is made less than $(\pi/2)-\theta_2$ because some of the available bandwidth present in the individual even- and odd-mode networks is wasted. The zeros of the two networks become less coincidental, and in extreme cases may even occur outside the coupler passband.

Theoretical Results for Five-Branch Couplers

In practice, however, the directivity and VSWR deteriorate only slowly as θ_1 is decreased, whereas the impedance level undergoes rapid improvement. The effect is illustrated in Table I, which gives the design and performance of several Zolotarev branch-guide couplers, each having the same fundamental specification in respect of the number of branches, midband coupling, and bandwidth. The specification chosen, namely $n=5$, $\lambda_{g0}/\lambda_{g2}=1.2$, and 3.07-dB coupling, is given approximately in [1], where for 3.0-dB midband coupling the directivity is 45.6 dB and the VSWR is 1.008. These are the

TABLE I
PERFORMANCE AND ELEMENT VALUES OF ZOLOTAREV BRANCH-GUIDE COUPLERS

Zolotarev Lower angle θ_1 (degrees)	Directivity (dB)	VSWR	Branch Guide Impedances			Main line Impedances	
			a_1	a_2	a_3	b_1	b_2
36	45.6	1.008	0.1058	0.5747	1.2207	1.2121	1.8053
33	42.9	1.010	0.1309	0.4820	0.7930	1.1176	1.3915
30	42.0	1.011	0.1438	0.4296	0.6153	1.0604	1.1896
27	40.5	1.013	0.1513	0.4010	0.5329	1.0243	1.0808
24	38.0	1.018	0.1547	0.3820	0.4876	0.9998	1.0154

$N=5$; midband coupling = 3.07 dB; $\lambda_{g0}/\lambda_{g2}=1.2$; coupling at band edges = 2.54 dB; Zolotarev upper angle $\theta_2=54°$.

same values given by the 3.07-dB design with $\theta_1=36°$ (the electrical optimum) in Table I. The design given in [1] is slightly better physically than the Zolotarev design with $\theta_1=36°$, for the largest main line impedance is 1.681 compared with 1.8053. However, the advantage given in the Zolotarev design by the flexibility in the choice of θ_1 is seen by inspection of the other designs in Table I. Thus reducing θ_1 from 36° to 30° gives only 3.6-dB reduction in directivity and negligible increase in VSWR, but the central main line impedance drops to 1.1896. This is accompanied by an increase in the smallest branch guide from 0.1058 to 0.1438 and a reduction in the largest branch-guide impedance from 1.2207 to 0.6153. The 30° coupler can be accurately designed in practice. The electrically optimum 36° coupler probably cannot. Similar results are obtained for any other values of n, bandwidth, and coupling.

The design process requires access to a computer program that forms the Zolotarev prototype networks. For a given number of branches and upper and lower electrical lengths θ_1 and θ_2, the problem is then to choose the prototype VSWR so that the desired value of midband coupling is obtained. Since the midband coupling is an elementary function of the transfer matrix parameters for either the even- or odd-mode prototype networks, and since these parameters are formed directly in the synthesis technique, it is simple to print out the value of midband coupling for any value of prototype VSWR. It is possible to home on to the correct value of VSWR rather quickly, or this may be performed automatically within the program by simple iteration.

PRACTICAL RESULTS WITH APPLICATION TO MATCHED POWER DIVIDER DESIGN

Some of the most interesting results of the new theory were obtained when it was applied to design branch-guide couplers used in a complex matched power divider network. This was constructed in reduced height WR284 waveguide having internal dimensions of 2.840×0.400 in. It should be noted immediately that the reduced height makes it slightly simpler to realize tight coupling because the absolute internal impedance level is reduced. However, this makes no difference to the relative impedance levels within the coupler, and the problem of central branches breaking into one another remains. Results equally as good as those presented here have been obtained with couplers constructed in normal height waveguide.

The power divider consisted of 23 subassemblies of 13 distinct types each having three-, four-, or five-way power division. The output port couplings were nonstandard, e.g., 3.59, 4.95, 7.85, and 11.08 dB in a typical case. It was necessary to design no less than 14 different 4-port directional couplers having couplings varying from 1.52 dB through 8.51 dB. The tolerance on power division for the power dividers was dependent on the coupling, but for the tightest coupled ports of around 4-dB nominal coupling, the allowable deviation from nominal was ±0.25 dB with a coupling flatness of ±0.1 dB over the specified frequency band of 3.1–3.5 GHz. This was relaxed to ±1.0 dB and ±0.4 dB, respectively, for couplings above 13 dB. Restricted space limitations dictated the use of short slot or branch-guide couplers. The latter type was selected because currently it is the only type of waveguide directional coupler that can be designed by nonempirical means to have accurately predictable coupling and other characteristics [2].

Six of the individual branch-guide couplers were constructed and tested as individual units, and the results for coupling are shown in Fig. 4. The couplers consisted of four or five branches and were designed on the Zolotarev function theory using internal impedance optimization, as described in this paper. The measured results are in good agreement with the computed theory in most cases. The 6.16-dB coupler was later discovered to have been made with slightly inaccurate dimensions, which explained the coupling error of 0.25 dB.

The 4.3-dB and 3.59-dB couplers were measured using an accurate single-frequency technique, and they show the best agreement between theory and experiment. The other measurements were made at an independent laboratory using a computer-controlled network analyzer. Clearly these results suffer from a random error of approximately ±0.1 dB, since there can be no ripple of this magnitude in the coupling of a well-matched branch-guide coupler of high directivity. The

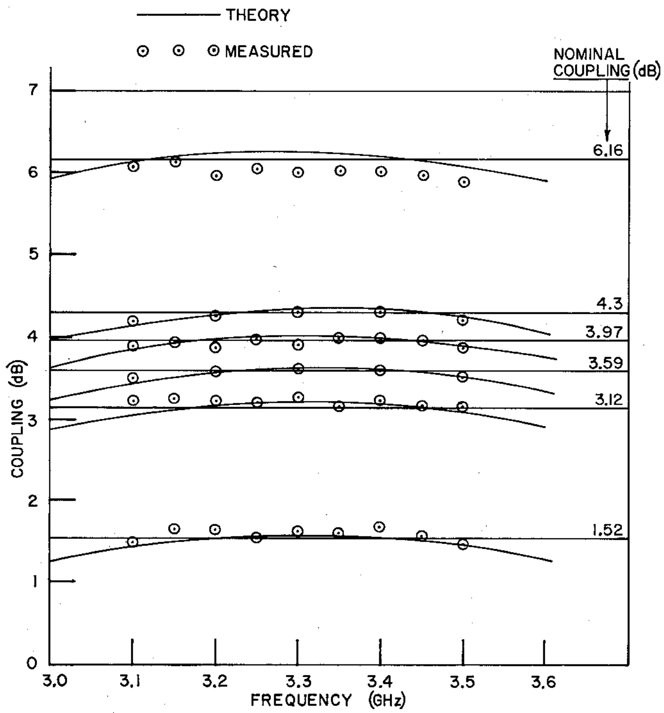

Fig. 4. Coupling characteristics of individual couplers.

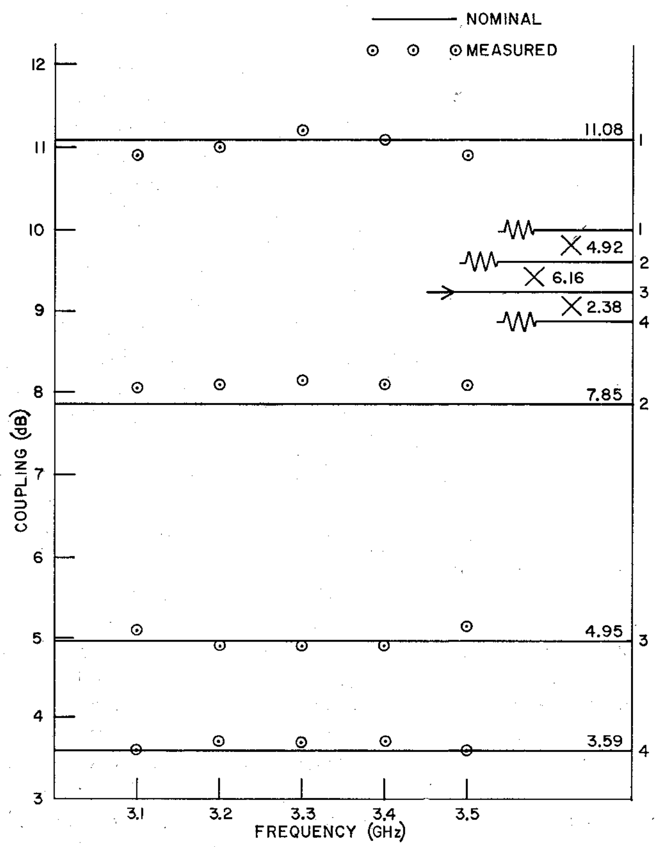

Fig. 5. Coupling of four-way power divider.

VSWR was generally better than 1.05 and the directivity greater than 20 dB, typically approximately 30 dB.

The measured characteristics of a four-way power divider are shown in Fig. 5. The insert indicates that the power divider consists of three branch-guide couplers of 2.38-, 4.92-,

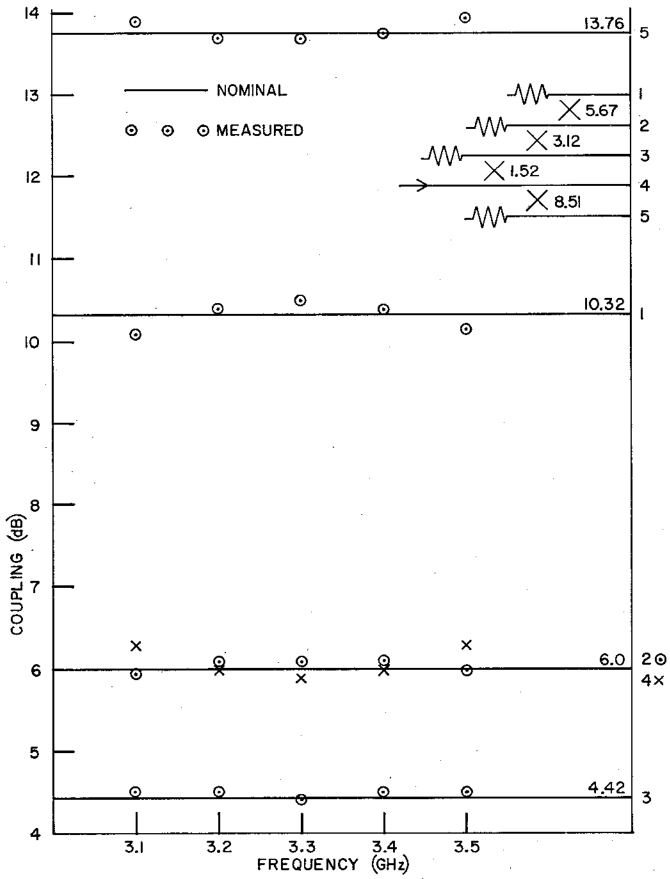

Fig. 6. Coupling of five-way power divider.

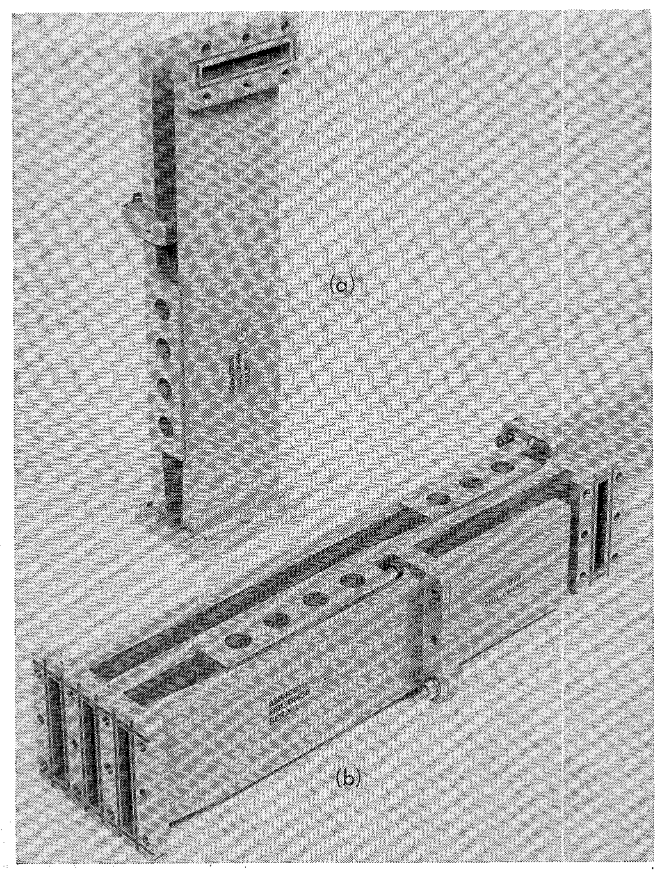

Fig. 7. (a) Branch-guide coupler. (b) Three-way power divider.

and 6.16-dB nominal coupling. The measured results at the four output ports are within the specified toleration deviations from nominal and flatness of coupling, which was the case for all of the various power divider assemblies. Similar characteristics for a five-way power divider are shown in Fig. 6. It is interesting to note how the coupling variation at any output port depends on the particular path taken by the signal. For example, this explains why the coupling to port 3 of the five-way power divider is so flat, the signal reaching this port by coupling "across" the 1.52-dB coupler but "through" the 3.12-dB coupler, hence being attenuated by couplings whose frequency characteristics tend to cancel. Ports 2 and 4 of this assembly each have 6-dB nominal coupling, and the shape of the coupling characteristics in these cases may be explained on a similar basis.

The couplers and power dividers were constructed in aluminum and their physical form and construction is indicated in Fig. 7. All 23 power dividers met specification with no empirical adjustments being required. This is an example of how precise computer design for components facilitates design of a complex assembly.

Conclusions

Branch-guide couplers having tight-coupling values may be designed directly, without cascading two or more couplers of looser coupling, by using the new design theory based on Zolotarev functions. This method enables the internal impedance levels of the main lines and branch guides to be optimized so that they may be physically constructed, and this may be carried out with very little theoretical deterioration in directivity and VSWR. Almost perfect correlation between theory (taking junction effects into account) and experiment has been obtained in measurements performed on over 100 branch-guide couplers. The design of a complex matched power divider network direct from computer programs to hardware was described with complete agreement between computations and measured results.

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A Least-Squares Boundary Residual Method for the Numerical Solution of Scattering Problems

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***Abstract*—An explicit least-squares criterion is put forward as an alternative to the point-matching method of numerically solving scattering problems. While being an established method of functional approximation, it has been largely ignored in numerical approaches to electromagnetic scattering.**

In contrast to point matching, the least-squares approach has a rigorous proof of convergence. An electric/magnetic weighting factor is found useful in optimizing convergence. Finally, it allows use of perhaps the fastest and most compact matrix inversion algorithm.

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I. Introduction

A NEW NUMERICAL approach is proposed for solving problems of electromagnetic wave scattering. Its justification and potential is described mainly by comparison with the point-matching (or collocation) method, which has received much attention lately.

In point matching, Fourier matching, and the proposed least-squares approaches, advantage is taken of the fact that one can easily satisfy the differential equations of the problem. By using, over each of a number of regions, truncated series from complete expansions, the problem is reduced to satisfying boundary conditions (and possibly edge or radiation